Province of the
EASTERN CAPE
EDUCATION

MATHEMATICS P1

COMMON TEST

JUNE 2014

NATIONAL
SENIOR CERTIFICATE

GRADE 12

Marks: 125

Time: 2½ hours

N.B: This question paper consists of 7 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of this question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 5x - 6 = 0$ (3)

1.1.2 $-3x^2 + 4x + 2 = 0$ (3)

1.1.3 $\dfrac{x^2}{x+2} \leq 0$ (3)

1.1.4 $2^{x+3} - 3.2^{x-1} = 104$ (5)

1.2 Simplify, without the use of a calculator: $\sqrt{72x^2} - \sqrt{98x^2} + 2\sqrt{288x^2}$ (3)

1.3 Solve for x and y where: $2x - y = 2$ and $y = (x-2)(x-1)$ (6)

[23]

QUESTION 2

2.1 Given the sequence: 2 ; 5 ; 8 ; …

2.1.1 If the pattern continues, then write down the next two terms. (1)

2.1.2 Prove that none of the terms of this sequence are perfect squares. (5)

2.2 1; 3; 5 are the first three terms of the **first differences** of a quadratic sequence. The 7^{th} term of the quadratic sequence is 35.

2.2.1 Determine the 6^{th} and 5^{th} terms of the quadratic sequence. (4)

2.2.2 Determine the n^{th} term of the quadratic sequence. (5)

2.3 Prove that the sum to n terms of a geometric sequence is given by:

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$ (4)

2.4 Calculate the value of n if:

$$\sum_{k=1}^{n} 2(3)^{k-1} = 531440$$ (5)

[24]

QUESTION 3

Given $f(x) = -(x+2)^2 + 6$ and $g(x) = 2^{-x} + 1$.

3.1 Draw graphs of f and g on the same set of axes. Clearly show the intercepts with both axes, as well as the asymptote(s) where applicable. (8)

3.2 Write down the value(s) of t if $f(x) = t$ has:

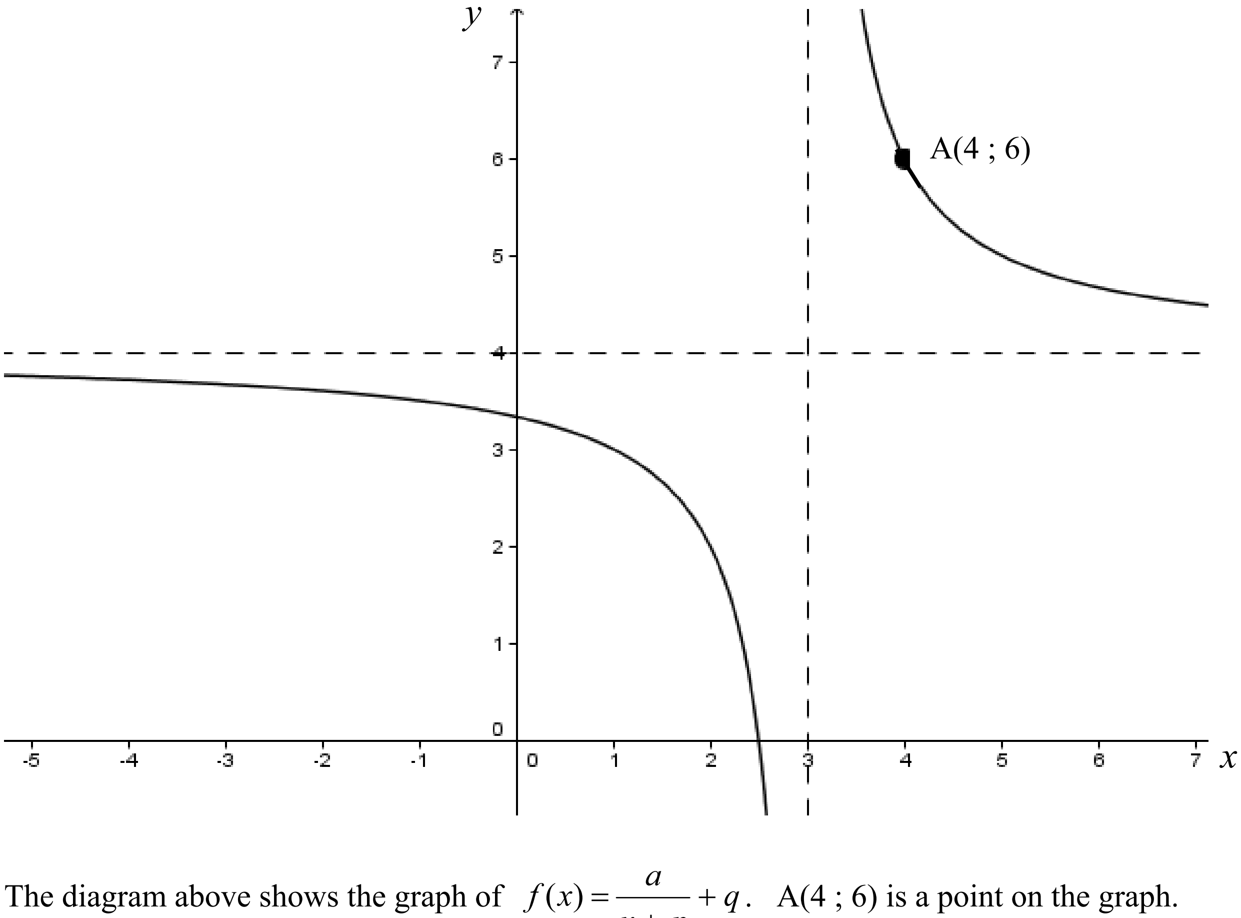
3.2.1 equal roots. (2)

3.2.2 one root equal to 0. (2)

3.3 Write down the equation of the asymptote of h if $h(x) = g(x) + 1$. (2)

[14]

QUESTION 4



The diagram above shows the graph of $f(x) = \dfrac{a}{x+p} + q$. A(4 ; 6) is a point on the graph.

4.1 Determine the value(s) of a, p, and q. (4)

4.2 Write down the range of g if $g(x) = f(x) - 2$. (2)

4.3 If the graph of f is symmetrical with respect to the line $y = x + c$, determine the value of c. (3)

[9]

QUESTION 5

5.1 Given: $f(x) = log_5 x$

Determine f^{-1}. (2)

5.2 Given $h(x) = x^2$

5.2.1 Determine the inverse of h in the form $y = \ldots..$ (2)

5.2.2 Give a reason why the inverse of h is not a function. (2)

5.2.3 Write down TWO ways in which you can restrict the domain of h so that its inverse is a function. (2)

5.2.4 Hence, sketch the graphs of the function h^{-1}. (4)

5.2.5 Determine the value(s) of x for which $h^{-1}(x) \leq 2$. (2)

[14]

QUESTION 6

6.1 Determine the derivative of $f(x) = x^3$ from first principles. (5)

6.2 Calculate the derivative of the following:

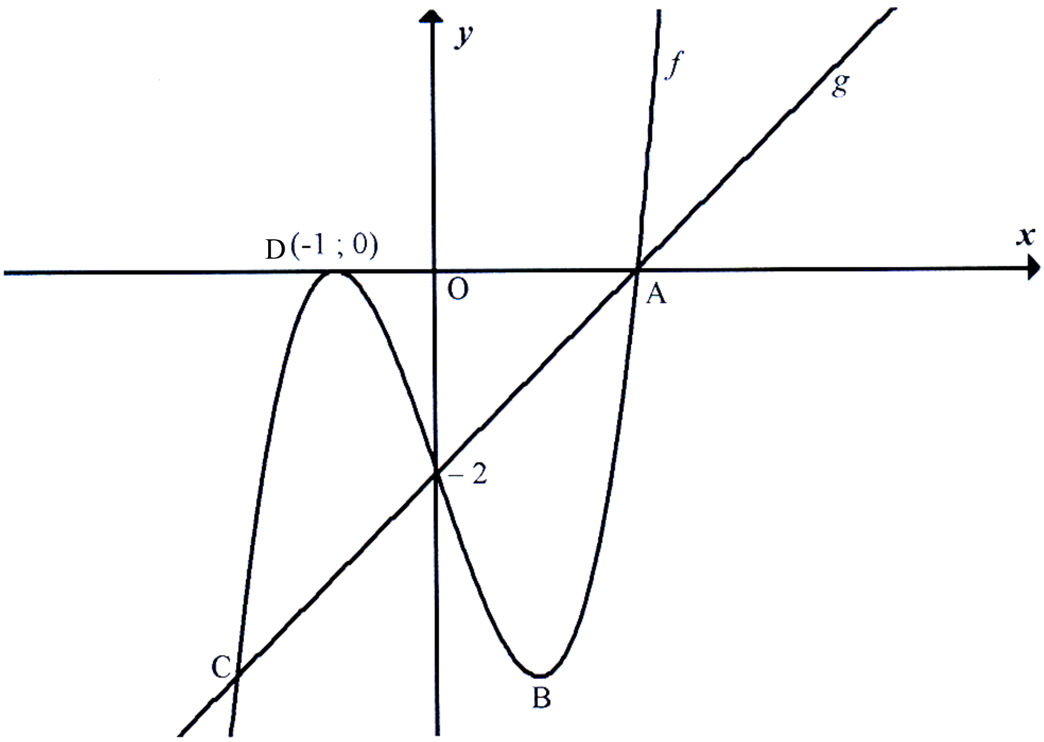
6.2.1 $x^2\left(1 - \frac{1}{x}\right)$ (4)

6.2.2 $h(x) = \frac{\sqrt[3]{x^2} - 3x}{\sqrt{x}}$ (4)

[13]

QUESTION 7

The graph below represents the functions f and g with $f(x) = ax^3 - cx - 2$ and $g(x) = x - 2$.
A and D(– 1; 0) are the x-intercepts of f. The graphs of f and g intersect at A and C.



7.1 Determine the coordinates of A. (2)

7.2 Show by calculation that $a = 1$ and $c = 3$. (5)

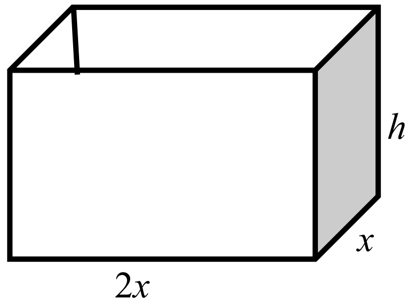
7.3 Determine the coordinates of B, a turning point of f. (4)

7.4 Determine the x-coordinate of the point of inflection of f. (2)

7.5 Write down the values of k for which $f(x) = k$ will have only ONE root. (3)

7.6 Write down the values of x for which $f'(x) < 0$. (2)
[18]

QUESTION 8



A crate used on vegetable farms in the Ponono Area is in the form of a rectangular prism which is open on top. It has a volume of 1 cubic metre. The length and the breadth of its base is 2x, and x metres respectively. The height is h metres. The material used to manufacture the base of this container costs R200 per square metre and for the sides, R120 per square metre.

8.1 Express h in terms of x. (2)

8.2 Show that the cost , C, of the material is given by:

$C(x) = 400x^2 + 360x^{-1}$ (3)

8.3 Calculate the value of x for which the cost of the material will be a minimum and hence the minimum cost of the material. (5)

[10]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$